Examples & board definitions for part 5 of the "Model Checking and Games" lecture series

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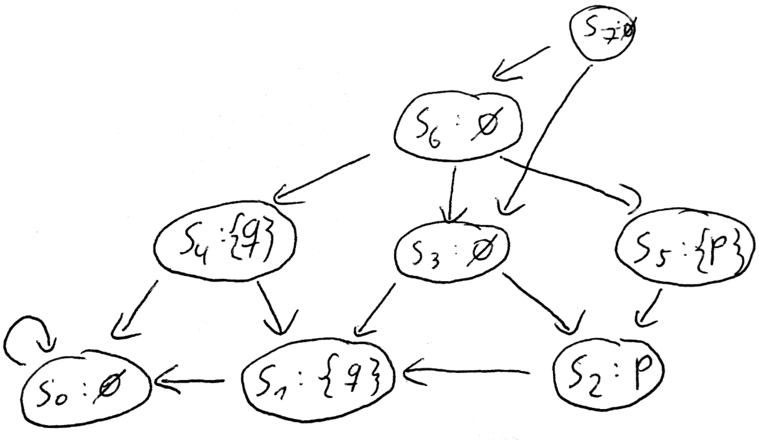
1 Example for evaluation of modal μ-calculus formula

We consider the following formula: $\mu X.(\Diamond p \cap \Diamond y) \cup \Box X$

We consider the following Kripke structure:



We evaluate:

- $\llbracket p \rrbracket_M = \{s_2, s_5\}$ (regardless of the choice of M)
- $\llbracket q \rrbracket_M = \{s_1, s_4\}$ (regardless of the choice of M)
- $\llbracket \Diamond p \rrbracket_M = \{s_3, s_5, s_6\}$ (regardless of the choice of M)
- $\llbracket \Diamond q \rrbracket_M = \{s_2, s_3, s_4, s_6\}$ (regardless of the choice of M)
- $\llbracket \Diamond p \cap \Diamond q \rrbracket_M = \{s_3, s_6\}$ (regardless of the choice of M)
- $\llbracket \mu^0 .X.(\Diamond p \cap \Diamond y) \cup \Box X \rrbracket_\varnothing = \varnothing$
- $\llbracket \mu^1 .X.(\Diamond p \cap \Diamond y) \cup \Box X \rrbracket_\varnothing = \llbracket (\Diamond p \cap \Diamond y) \cup \Box X \rrbracket_{\{X \mapsto \varnothing\}} = \llbracket (\Diamond p \cap \Diamond y) \rrbracket_{\{X \mapsto \varnothing\}} \cup \llbracket \Box X \rrbracket_{\{X \mapsto \varnothing\}} = \{s_3, s_6\}$
- $\llbracket \mu^2 .X.(\Diamond p \cap \Diamond y) \cup \Box X \rrbracket_\varnothing = \llbracket (\Diamond p \cap \Diamond y) \cup \Box X \rrbracket_{\{X \mapsto \{s_3, s_6\}\}} = \llbracket (\Diamond p \cap \Diamond y) \rrbracket_{\{X \mapsto \{s_3, s_6\}\}} \cup \llbracket \Box X \rrbracket_{\{X \mapsto \{s_3, s_6\}\}} = \{s_3, s_6, s_7\}$

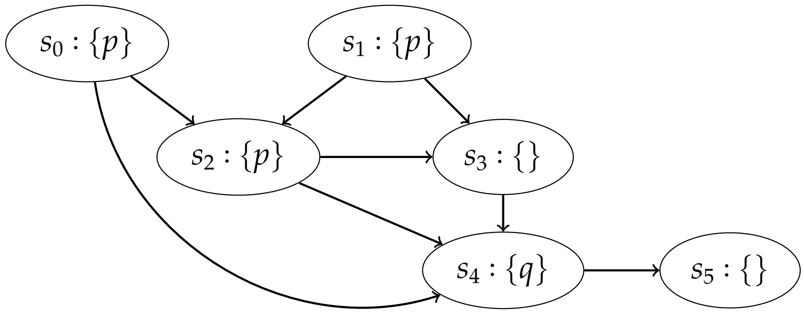
- $[\![\mu^3.X.(\Diamond p \cap \Diamond y) \cup \Box X]\!]_\varnothing = [\![(\Diamond p \cap \Diamond y) \cup \Box X]\!]_{\{X \mapsto \{s_3,s_6,s_7\}\}} = [\![(\Diamond p \cap \Diamond y)]\!]_{\{X \mapsto \{s_3,s_6,s_7\}\}} \cup [\![\Box X]\!]_{\{X \mapsto \{s_3,s_6,s_7\}\}} = \{s_3, s_6, s_7\}$
- Hence, $[\![\mu.X.(\Diamond p \cap \Diamond y) \cup \Box X]\!]_\varnothing = \{s_3, s_6, s_7\}$.

2 CTL example

Let us consider the CTL formula $\mathsf{E}(p \mathcal{U}\, \mathsf{AX} q)$. We can translate it to the following μ-calculus formula to evaluate from which states in a Kripke structure the CTL formula holds:

$$\mu X. \Box\, q \vee (p \wedge \Diamond X)$$

As an example, let us evaluate the μ-calculus formula for the following Kripke structure:



(The solution will be on the whiteboard)